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Single Specimen Determination of Young's and Bulk Moduli

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Synopsis

A new method is presented for the rapid and consecutive determination of the Young's and bulk moduli of polymeric materials. Usually these properties are determined by separate measurements. However, by using an undersized specimen in a standard compressibility device it has been found possible consecutively to determine both moduli on the same specimen in a single test procedure. The initial loading of the undersized specimen results in a decrease in length and an increased diameter. From these changes Young's modulus may be calculated. Once the bore of the tester is filled, the application of additional pressure results in a decrease in the volume of the polymer, from which the bulk modulus may be calculated. Both of these determinations may be made within minutes, and the values of the moduli are in agreement with published values.

Young's modulus and the bulk modulus are both important fundamental properties of the solid state. Generally, they are determined by separate and independent measurements. Or the bulk modulus is calculated from other moduli values. However, there appears to be no one method by which both moduli can be measured directly and simply on the same specimen in the same apparatus. It is the purpose of this paper to describe a new rapid isothermal technique for accomplishing such measurements.

The modulus of a Hookean solid under load is defined as the ratio of the applied stress to the resulting strain. Young's modulus E is rather easily determined in tension and in compression.[1] In both cases it is the simple unit load divided by the unit deformation of the specimen. Bulk modulus B is taken as the ratio of hydrostatic pressure to volume strain, and is generally more difficult to determine than is E. In either instance, different test equipments and different specimens are required for each of these measurements.

We have recently found that a Matsuoka-Maxwell type tester[2] can be used to determine both E and B consecutively on the same polymer specimen in a single test procedure. Briefly, the apparatus consists of a hardened steel inner bushing, the inner surface of which is lapped and polished to 0.635 cm. (0.2500, +0.0001, −0.0000 in.) in diameter. This inner bushing is tightly fitted into an outside steel casing.

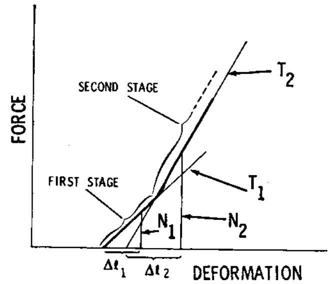

Fig. 1. Stress–strain plot for polystyrene.

In preparation for an experimental run, a polymer specimen 7.62 cm. (3 in.) long and 0.630 cm. (0.248 in.) in diameter is inserted into the bore of the tester. Care is taken to assure that the specimen is 0.005 cm. smaller in diameter than the bore. Then two case-hardened steel plungers 4.128 cm. (1.625 in.) long and 0.635 cm. (0.2500 + 0.0000, −0.0001 in.) in diameter are inserted into the open ends of the bore. This assembly is next placed in a testing machine and the specimen is loaded by pressing down on the steel plungers. The load is applied at a rate of 0.025 in./min. up to any arbitrary level desired. Additional details on this technique have been previously given.[3]

If the polymer specimen exactly fills the bore of the tester, pressure will cause a decrease in the volume of the polymer and the bulk modulus can be determined. However, when the diameter of the specimen is made slightly smaller, application of pressure will initially result in a decrease in length and an increase in diameter. These changes will continue to occur until the bore of the tester becomes filled. Then additional application of pressure results in a decrease in the volume of the specimen.

A typical stress–strain plot recorded for an undersized polystyrene specimen is shown in Figure 1. Two stages will be noted. From the first stage, Young's modulus can be calculated. This is done by drawing a tangent, T_1, to the first stage of the plot. A normal N_1 is then dropped from a point on T_1 to the abscissa. The distance between its intersection and that of T_1 describes a characteristic specimen deformation Δl_1 for stage 1, for a force equivalent to the height of N_1. E is then calculated by using eq. (1):

$$E = \frac{F_1/A}{\Delta l_1/l_0} \tag{1}$$

where l_0 is the original specimen length, F_1 is the applied force, and A is original area of the plunger which transmits the load to the specimen.

The second stage of the plot develops at the time when the specimen is no longer capable of expanding in the radial direction, due to the constriction of the bore. Under these conditions, the following relations are valid:

$$B = \frac{F_2/A}{\Delta V/V_0} = \frac{F_2/A}{\Delta l_2/l_0} \tag{2}$$

Thus, in a similar manner as before, the bulk modulus is obtained by drawing tangent T_2, dropping normal N_2 to the abscissa determining Δl_2, and then using eq. (2) for the calculation of B.

In general, this technique is applicable to polymeric solids which can be hydrostatically compressed in a piston-cylinder apparatus, and having Poisson's ratios of about 0.35 and higher. Typical values of Young's and bulk moduli for polymers, obtained with this technique are given in Table I.

TABLE I

Material	Modulus, dyne/cm.^{2} $\times 10^{-10}$	
	Young's	Bulk
Polyepoxide polymer	3–4	7
Polystyrene	3.6	5.4
Poly(methyl methacrylate)	3.2	5.1
Nylon 66	1.8	8.1
Polychlorotrifluoroethylene	1.9	5.2

Poisson's ratios calculated from the moduli given in Table I are most reasonable when compared to available data.

If it is desired to determine either moduli as a function of temperature, this is also easily accomplished by placing heating coils around the tester and measuring the increase in temperature by means of two embedded thermocouples.[2,3] When determining Young's modulus at elevated temperatures, the diameter of the specimen must be carefully chosen so that even at the highest temperature it will be smaller than the bore of the tester by 0.005 cm. (0.002 in.).

We conclude that this technique offers the following advantages over existing methods: (*1*) both moduli can be determined consecutively on the same specimen, (*2*) both moduli can be determined rapidly and with a minimum of calculations, (*3*) all data are determined under isothermal conditions, and (*4*) the same apparatus may be employed in both determinations. A complete discussion of this technique will be published later.

References

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